

# Controlling Tri-level Center-Split Power Quality Compensator by 3-Dimensional Space Vector Modulation

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**Abstract**—In the past researches, strategies based on Hysteresis Control are proposed for controlling tri-level 3-leg center-split inverter, which can be used as a shunt power quality compensator for three-phase four-wire system. In this paper, results indicate that the complicated 3-dimensional (3-D) space vector allocation of the tri-level center-split inverter can be simplified into six space vector regions of two-level center-split inverter. Thus, based on the 3-D space vector modulation (3DSVM) control strategy for two-level center-split inverter, the novel 3DSVM control strategy for tri-level 3-leg center-split inverter is proposed. Different from the conventional 2-dimensional space vector modulation (2DSVM), the zero-axis component should be of a major concern in this novel 3DSVM since 3-D space vectors are located in different horizontal planes and each vector contributes to zero-axis compensation. When the zero-axis component is considered, the neutral current of the three-phase four-wire system can be compensated simultaneously with three-phase current harmonics. Simulation results are given to show the validity of the proposed control strategy.

**Keywords**—Power Quality, Three-Phase Four-Wire System, Tri-level Center-Split Inverter, 3DSVM

## I. INTRODUCTION

Due to the development of “Custom Power” concept, three-phase four-wire system plays a very important role in the Distribution Site. After a wire is connected between the mid-point of the dc linked capacitors and the ac neutral wire, the conventional tri-level Neutral Point Clamped inverter for three-phase three-wire system can be used as a shunt power quality compensator for three-phase four-wire system.

However, there is seldom attention has been paid to the control strategies of tri-level inverter for three-phase four-wire system. Only Sign Cubic and Cylindrical Coordinate Hysteresis control strategies are discussed [1] [2]. But, there is no report on the space vector modulation control strategy yet. In this paper a novel 3-Dimensional Space Vector PWM (3DSVM) is proposed for controlling tri-level 3-leg center-split inverter for three-phase four-wire system. This novel method is based on 3DSVM for two-level center-split inverter. Firstly, the 3D space vector allocation of tri-level 3-leg center-split inverter is introduced. Then the basic principle of the proposed 3DSVM is discussed in detail, which will mainly focus on the division of tri-level space vector diagram and the correction of the reference voltage vector. Results indicate

that the complicated 3-D space vector allocation of the tri-level center-split inverter can be simplified into six space vector regions of two-level center-split inverter. And after the reference voltage vector is corrected, all the remaining calculations of switching time for the 3DSVM are done as 3DSVM for 2-level inverter. In the last part of the paper, the proposed 3DSVM is used to control a tri-level 3-leg center-split inverter which is used as a shunt power quality compensator for three-phase four-wire system. Simulation results are given to show that current harmonics and neutral current can be compensated simultaneously when the proposed 3DSVM strategy is employed.

## II. BASIC PRINCIPLES

### A. Definition of 3-D Space Vectors

The tri-level 3-leg “Center-Split” Inverter structure [1][2] is the basic configuration for the discussion of 3-dimensional Pulse-Width Modulation in this paper. Fig. 1 shows a tri-level center-split inverter as a shunt connected power quality compensator for three-phase four-wire system. It is assumed that the upper-leg and the lower-leg capacitor voltages in Fig. 1 are the same:  $V_{dc1} = V_{dc2} = V_{dc}$ . The switching function can be defined as:

$$S_j = \begin{cases} 1, & \text{when } T_{1j} \text{ and } T_{2j} \text{ are closed} \\ 0, & \text{when } T_{2j} \text{ and } T_{3j} \text{ are closed} \\ -1, & \text{when } T_{3j} \text{ and } T_{4j} \text{ are closed} \end{cases} \quad j=a,b,c \quad (1)$$

Accordingly, the output voltage of one leg of the inverter can be expressed as:

$$v_j = V_{dc} * S_j \quad j=a, b, c \quad (2)$$

The instantaneous voltage vector in a-b-c frame can be described as shown in equation (3). By using the  $\alpha$ - $\beta$ -0 transformation principle as shown in equation (4), the instantaneous voltage vector of the 3-leg inverter in  $\alpha$ - $\beta$ -0 frame can be given as equation (5).

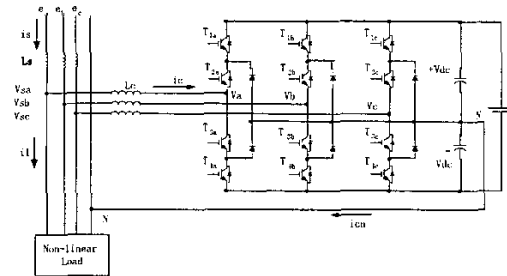


Fig. 1 3-Phase 4-Wired Three-Level Converter

$$\vec{v} = \sqrt{\frac{2}{3}}(v_a + \alpha \cdot v_b + \alpha^2 \cdot v_c) \quad (3)$$

, where  $\alpha = e^{j\frac{2\pi}{3}}, \alpha^2 = e^{-j\frac{2\pi}{3}}$

$$\begin{bmatrix} v_a \\ v_b \\ v_0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \quad (4)$$

$$\vec{v} = V_{dc} \left[ \sqrt{\frac{2}{3}} S_a \cdot \vec{n}_a + \frac{1}{\sqrt{2}} S_b \cdot \vec{n}_b + \frac{1}{\sqrt{3}} S_0 \cdot \vec{n}_0 \right] \quad (5)$$

where  $S_a = S_a - \frac{1}{2} S_b - \frac{1}{2} S_c$ ,

$$S_b = S_b - S_c,$$

$$S_0 = S_a + S_b + S_c.$$

In 3-Dimensional aspect, except conventional a-b-c coordinates, space vector allocation can be expressed in Rectangular coordinates as shown in Fig.2. It can be seen from Fig. 2 that  $\{\vec{n}_a, \vec{n}_b, \vec{n}_0\}$  form a basis and they are orthogonal to each other i.e.  $\vec{n}_0 \cdot \vec{n}_a = \vec{n}_0 \cdot \vec{n}_b = \vec{n}_a \cdot \vec{n}_b = 0$ .

The instantaneous voltage vector in  $\alpha$ - $\beta$ -0 frame can be expressed as:

$$\vec{v} = v_a \vec{n}_a + v_b \vec{n}_b + v_0 \vec{n}_0 \quad (6)$$

where  $v_a = V_{dc} \cdot \sqrt{\frac{2}{3}} S_a$ ,  $v_b = V_{dc} \cdot \frac{1}{\sqrt{2}} S_b$ ,  $v_0 = V_{dc} \cdot \frac{1}{\sqrt{3}} S_0$ .

### B. 3-D Space Vectors

#### B.1 Two-Level 3-D Space Vectors

In the two-level conventional Pulse Width Modulation techniques, the voltage space vectors can be illustrated in  $\alpha$ - $\beta$  frame, as shown in Fig. 3. The number of states in two-level system is seven although there are eight available vectors altogether. There are totally eight voltage vectors with six directional vectors and two zero vectors. In past researches, these two zero vectors were utilized in optimizing the switching losses. But, in 3DPWM, these 2 vectors dedicated as the zero-axis voltage components in positive and negative directions respectively. Table 1 summarized the parameters describing the two-level 3DPWM voltage vectors in a-b-c and  $\alpha$ - $\beta$ -0 Coordinates.

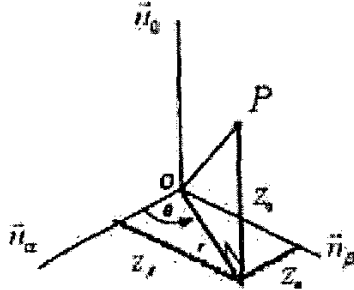


Fig. 2 Rectangular Coordinate

Table 1 Two-level 3DPWM Voltage Vector's Parameters

	$S_a$	$S_b$	$S_c$	$S_a$	$S_b$	$S_0$

$\vec{V}_1$	1	-1	-1	2	0	-1
$\vec{V}_2$	1	1	-1	1	2	1
$\vec{V}_3$	-1	1	-1	-1	2	-1
$\vec{V}_4$	-1	1	1	-2	0	1
$\vec{V}_5$	-1	-1	1	-1	-2	-1
$\vec{V}_6$	1	-1	1	1	-2	1
$\vec{V}_{00p}$	1	1	1	0	0	3
$\vec{V}_{00n}$	-1	-1	-1	0	0	-3

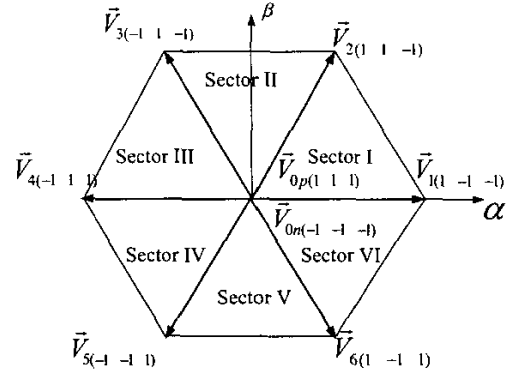


Fig. 3 Two-level Voltage Space Vector's allocation in  $\alpha$ - $\beta$  frame

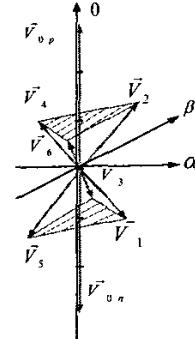


Fig. 4 Two-level 3D Voltage Vectors

The two-level voltage vector's allocation is better to be described in  $\alpha$ - $\beta$ -0 3-D aspect, as shown in Fig. 4. The Vectors  $\{\vec{v}_2, \vec{v}_4, \vec{v}_6\}$  and  $\{\vec{v}_1, \vec{v}_3, \vec{v}_5\}$  lie on the different horizontal planes. It is clear that the conventional 2D PWM voltage vector's allocation is just the top view of 3DPWM. Actually, 2D PWM technique is a subset of 3D PWM.

#### B.2 Tri-Level 3-D Space Vectors

The number of vectors, which can be implemented in the tri-level center-split VSI, is 27. These can be categorized into four different vectors: large-, mid-, small-, and zero-vectors. The conventional picture describing the 3-level voltage vector's allocation is shown in Fig. 5. The conventional 3-level PWM techniques have 27 states and 19 available vectors. The number of states is not equal to the number of vectors. However, in 3DPWM, the number of states can be equal to the number of available vectors.

In Fig. 6, 3-Level Voltage Vector's Allocation in 3D aspect  $\{\bar{n}_\alpha, \bar{n}_\beta, \bar{n}_0\}$  is shown. In fact, when the zero-axis  $\bar{n}_0$  is considered, there are seven voltage levels or units in zero-axis such as  $\{-3, -2, -1, 0, 1, 2, 3\}$ . In 2 positive units of zero-axis,  $\bar{n}_0$ , small vectors  $\{\bar{V}_{02p}, \bar{V}_{04p}, \bar{V}_{06p}\}$  are allocated on this level. On the other hand, the combinations of small vectors and large vectors  $\{\bar{V}_{01p}, \bar{V}_2, \bar{V}_{03p}, \bar{V}_4, \bar{V}_{05p}, \bar{V}_6\}$  are allocated to 1 positive unit of the zero-axis shown in Fig. 6. All the Medium vectors  $\{\bar{V}_{12}, \bar{V}_{23}, \bar{V}_{34}, \bar{V}_{45}, \bar{V}_{56}, \bar{V}_{61}\}$  are allocated to the zero unit of the zero-axis or  $\alpha$ - $\beta$  horizontal plane. Vice versa, the vectors  $\{\bar{V}_1, \bar{V}_{02n}, \bar{V}_3, \bar{V}_{04n}, \bar{V}_5, \bar{V}_{06n}\}$  and  $\{\bar{V}_{01n}, \bar{V}_{03n}, \bar{V}_{05n}\}$  are allocated to negative unit's levels of zero-axis respectively.

Tables 2, 3, 4 and 5 are representing all the parameters in 3-D Coordinates for Zero Vectors, Small Vectors, Medium Vectors and Large Vectors respectively.

Table 2 Zero Vectors

	$S_a$	$S_b$	$S_c$	$S_\alpha$	$S_\beta$	$S_0$
$\bar{V}_{000}$	0	0	0	0	0	0
$\bar{V}_{00p}$	1	1	1	0	0	3
$\bar{V}_{00n}$	-1	-1	-1	0	0	-3

Table 3 Small Vectors

	$S_a$	$S_b$	$S_c$	$S_\alpha$	$S_\beta$	$S_0$
$\bar{V}_{01p}$	1	0	0	1	0	1
$\bar{V}_{01n}$	0	-1	-1	1	0	-2
$\bar{V}_{02p}$	1	1	0	0.5	1	2
$\bar{V}_{02n}$	0	0	-1	0.5	1	-1
$\bar{V}_{03p}$	0	1	0	-0.5	1	1
$\bar{V}_{03n}$	-1	0	-1	-0.5	1	-2
$\bar{V}_{04p}$	0	1	1	-1	0	2
$\bar{V}_{04n}$	-1	0	0	-1	0	-1
$\bar{V}_{05p}$	0	0	1	-0.5	-1	1
$\bar{V}_{05n}$	-1	-1	0	-0.5	-1	-2
$\bar{V}_{06p}$	1	0	1	0.5	-1	2
$\bar{V}_{06n}$	0	-1	0	0.5	-1	-1

Table 4 Medium Vectors

	$S_a$	$S_b$	$S_c$	$S_\alpha$	$S_\beta$	$S_0$
$\bar{V}_{12}$	1	0	-1	1.5	1	0
$\bar{V}_{23}$	0	1	-1	0	2	0
$\bar{V}_{34}$	-1	1	0	-1.5	1	0
$\bar{V}_{45}$	-1	0	1	-1.5	-1	0
$\bar{V}_{56}$	0	-1	1	0	-2	0
$\bar{V}_{61}$	1	-1	0	1.5	-1	0

Table 5 Large Vectors

	$S_a$	$S_b$	$S_c$	$S_\alpha$	$S_\beta$	$S_0$
$\bar{V}_1$	1	-1	-1	2	0	-1
$\bar{V}_2$	1	1	-1	1	2	1
$\bar{V}_3$	-1	1	-1	-1	2	-1
$\bar{V}_4$	-1	1	1	-2	0	1
$\bar{V}_5$	-1	-1	1	-1	-2	-1
$\bar{V}_6$	1	-1	1	1	-2	1

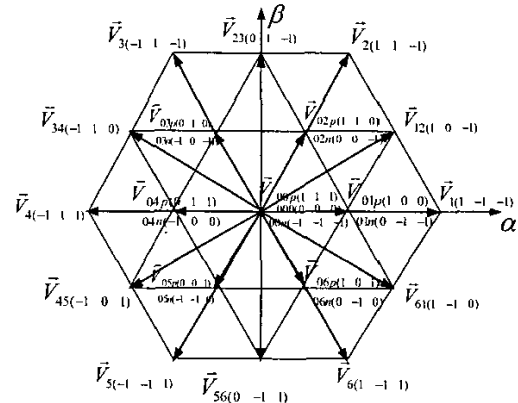


Fig.5 Tri-level voltage vector's allocation in  $\alpha$ - $\beta$  frame

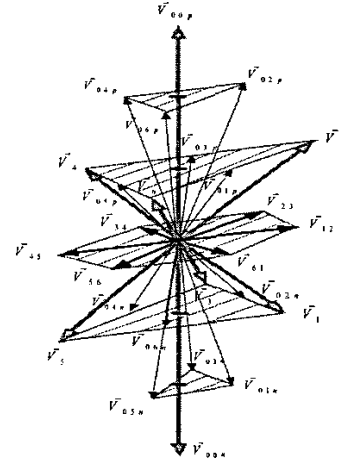


Fig.6 Tri-level 3D space vectors in  $\alpha$ - $\beta$ -0 frame

### III. 3DSVM CONTROL STRATEGY

#### A. Simplification of Tri-Level 3-D Space Vector Allocation

If the space-vector allocation of a tri-level inverter on  $\alpha$ - $\beta$  plane is considered, it is obvious that it is composed of six small hexagons as shown in Fig.7 [5]. And as shown in Fig.8, each small hexagon centers on an apex of the inner small hexagons and is the same as the space vector allocation of two-level inverter on  $\alpha$ - $\beta$  plane.

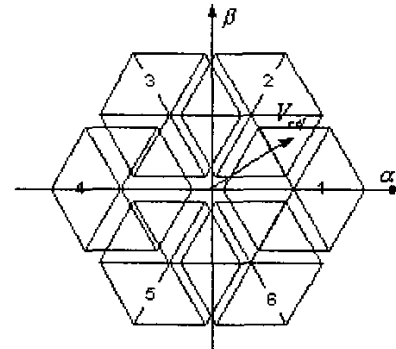


Fig.7 Division of tri-level space vector allocation

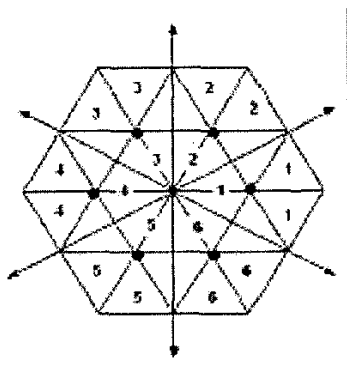


Fig. 8 Center-point position

In order to simplify the complicated 3-D space vector diagram of tri-level center-split inverter into a two-level one in  $\alpha$ - $\beta$ -0 frame, the parameters of each small hexagon's center point is defined and listed in Table 6. If these points are used as the center point of each hexagon, the 3-D space vector allocation in each hexagon is the same as that of two-level inverter, which indicates not only  $\alpha$ -,  $\beta$ -axis but also zero-axis relative position are matched. For example, if the hexagon 1 as shown in Fig.7 is considered, Fig.9 illustrates the transformation of voltage vector when the vectors origin moves from zero point (0,0,0) to the center-point of the selected hexagon, in which all the coordinates are represented by  $(S_\alpha, S_\beta, S_0)$  values. It can be seen from Fig. 9 that redundant vectors exist in  $\alpha$ - $\beta$  frame, i.e., at some position there are more than one space vector available. For conventional 2DSVM as discussed in [5], any vectors can be used. However, in the novel 3DSVM, the zero-axis position must be considered. Therefore, the space vectors which are circled by a rectangle in Fig. 9 are chosen so that the relative positions of these vectors are the same as that of two-level 3-D space vector allocation. Otherwise, the relative positions of these vectors will be different from that of two-level space vector allocation. And the 3-D space allocation of the space vector chosen in Fig.9 is shown in Fig.10. It can be seen from Fig.9 and Fig.10 that the relative position of vectors in hexagon 1 is the same as two-level 3-D space vector allocation both in  $\alpha$ - $\beta$  and  $\alpha$ - $\beta$ -0 frame. And considerations for the other five hexagons are the same.

### B. Simplified 3DSVM Control Strategy

The voltage-second reference of the 3DSVM technique of tri-level inverter is approximated by a sequence of voltage-second states as shown in (7) and (8). The vectors  $\vec{v}_x$ ,  $\vec{v}_y$  and  $\vec{v}_0$ , are chosen as they are the neighboring vectors of reference voltage vector.

$$\vec{V}_{ref} T_S = \vec{V}_x t_x + \vec{V}_y t_y + \vec{V}_0 t_0 + \vec{V}_{zero} t_{zero} \quad (7)$$

$$t_{zero} = T_S - t_x - t_y - t_0 \quad (8)$$

Table 6 Center voltage vector parameters

Hexagon	$S_\alpha$	$S_\beta$	$S_0$
1	1	0	-0.5

2	0.5	1	0.5
3	-0.5	1	-0.5
4	-1	0	0.5
5	-0.5	-1	-0.5
6	0.5	-1	0.5

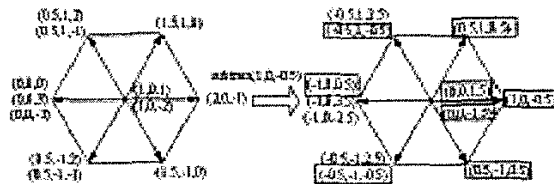


Fig. 9 Voltage allocation according to new center point

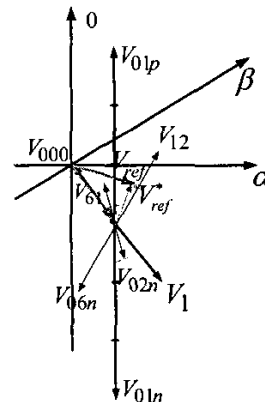


Fig. 10 Correction of reference voltage vector in  $\alpha$ - $\beta$ -0 frame

Based on the simplification of the space vector allocation of the tri-level center-split inverter, simplified 3DSVM for tri-level 3-leg center-split inverter is proposed.

The first step for the novel 3DSVM control strategy is correction of the reference voltage vector, which will be described in detail in this section. Firstly, according to the location of the reference voltage vector on  $\alpha$ - $\beta$  plane, a small hexagon area as shown in Fig.7 is chosen. Then, the reference voltage vector is corrected in  $\alpha$ -,  $\beta$ - and zero-axis respectively. The corrections of reference voltage vector in six different hexagon areas are summarized in Table 7. When the reference voltage vector is corrected, the origin of the reference voltage vector is moved to the center point of the selected hexagon as illustrated in Fig.10, where  $V_{ref}$  represents the original reference voltage vector and  $V_{ref}^*$  represents the new reference voltage vector obtained. And the correction of the reference voltage vector on  $\alpha$ - $\beta$  plane is illustrated in Fig. 11.

Table 7 Correction of reference voltage vector

Hexagon	$V_{ref\alpha}$	$V_{ref\beta}$	$V_{ref0}$
1	$V_{ref\alpha} - \sqrt{2/3} * V_{dc}$	$V_{ref\beta}$	$V_{ref0} + 0.5 * V_{dc} / \sqrt{3}$
2	$V_{ref\alpha} - 0.5 * \sqrt{2/3} * V_{dc}$	$V_{ref\beta} - V_{dc} / \sqrt{2}$	$V_{ref0} - 0.5 * V_{dc} / \sqrt{3}$
3	$V_{ref\alpha} + 0.5 * \sqrt{2/3} * V_{dc}$	$V_{ref\beta} - V_{dc} / \sqrt{2}$	$V_{ref0} + 0.5 * V_{dc} / \sqrt{3}$

4	$V_{ref\alpha} + \sqrt{2/3} * V_{dc}$	$V_{ref\beta}$	$V_{ref0} - 0.5 * V_{dc} / \sqrt{3}$
5	$V_{ref\alpha} + 0.5 * \sqrt{2/3} * V_{dc}$	$V_{ref\beta} + V_{dc} / \sqrt{2}$	$V_{ref0} + 0.5 * V_{dc} / \sqrt{3}$
6	$V_{ref\alpha} - 0.5 * \sqrt{2/3} * V_{dc}$	$V_{ref\beta} + V_{dc} / \sqrt{2}$	$V_{ref0} - 0.5 * V_{dc} / \sqrt{3}$

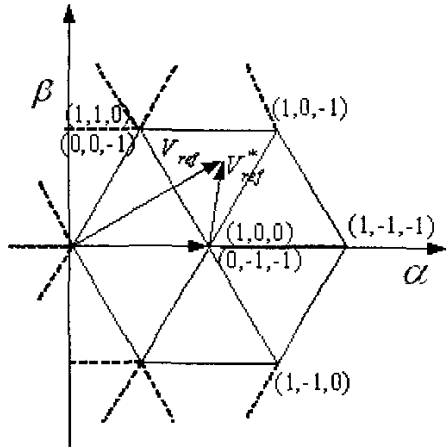


Fig. 11 Reference voltage vector correction on  $\alpha$ - $\beta$  plane

It can be seen from Fig. 10 and Fig. 11 that with the new reference voltage vector  $V_{ref}^*$ , the 3DSVM control for tri-level inverter is simplified into a small hexagon region. Based on simplification of the tri-level 3-D space vector allocation discussed hereinbefore, the space vectors relative positions in the small hexagon region are the same as a two-level 3DSVM case. Thus, after the reference voltage vector is corrected, the chosen of the neighboring vectors and the switching time calculation can be done as 3DSVM for two-level center-split inverter. However, one important thing needs to be mentioned here is that the length of the space vector in small hexagon area of tri-level inverter is only half of that in Fig.3 of two-level inverter. Thus when the switching time is calculated by using the equations of two-level 3DSVM, the switching time value obtained should be doubled.

#### IV. SIMULATION RESULTS

In this part, simulation is performed by Matlab/Simulink and the compensation system configuration is as shown in Fig.1. The 5 KHz switching frequency of the compensator is employed. The three-phase and neutral current at the load side is shown in Fig.12. Fig. 13 shows the source current compensation results when the proposed 3DSVM control strategy is used to control the tri-level 3-leg Center-Split inverter. The THD values of three-phase current before and after compensation are listed in Table 8. It can be seen from Fig.13 and Table 8 that the three-phase current harmonics and neutral current are compensated simultaneously.

Table 8 Current THD Value

Current THD	A	B	C
THD of load current	34.2%	34.17%	34.17%
Source current after compensation	3.38%	3.34%	3.34%

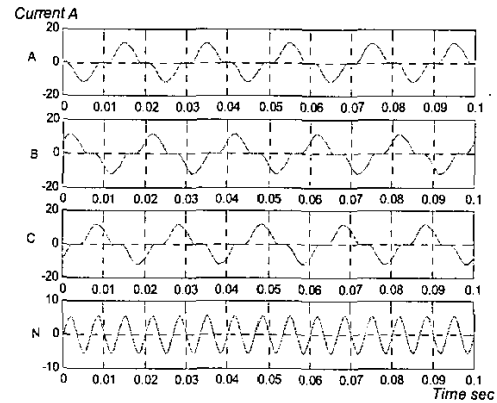


Fig.12 Load Current

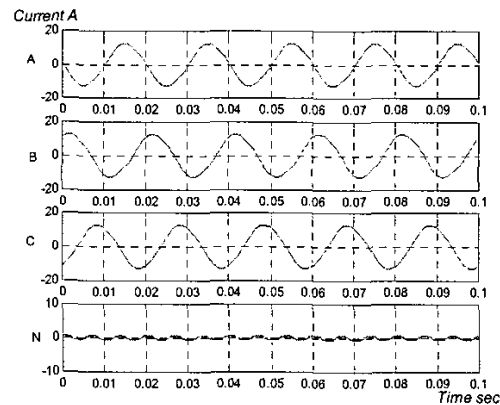


Fig. 13 Source Current after Compensation by 5KHz 3DSVM

#### V. CONCLUSION

In this paper a novel 3DSVM control strategy for tri-level 3-leg center-split inverter is proposed. Compared with simplified 2DSVM for conventional tri-level 3-leg inverter, there are totally 27 vectors corresponding to 27 switching states in the novel 3DPWM. And the consideration of zero-sequence component is implemented since it will contribute to the neutral current compensation for three-phase four-wire system. In the paper, results indicate that the complicated 3-D space vector allocation of the tri-level center-split inverter can be simplified into six space vector regions of two-level center-split inverter. So, in the simplified 3DSVM control strategy, after the reference voltage vector is corrected according to its location on  $\alpha$ - $\beta$  plane, the calculation of switching time can be done as 3DSVM for two-level inverter. The novel 3DSVM is applied to the tri-level 3-leg center-split inverter, which is used as a shunt power quality compensator for three-phase four-wire system. Simulation results are given to prove the validity of the proposed control strategy.

#### ACKNOWLEDGMENT

The authors would like to thank the Research Committee of University of Macau and Tsinghua University for their financial supports.

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