BCCB preconditioners for systems of BVM-based numerical integrators

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SUMMARY

Boundary value methods (BVMs) for ordinary differential equations require the solution of non-symmetric, large and sparse linear systems. In this paper, these systems are solved by using the generalized minimal residual (GMRES) method. A block-circulant preconditioner with circulant blocks (BCCB preconditioner) is proposed to speed up the convergence rate of the GMRES method. The BCCB preconditioner is shown to be invertible when the BVM is $A_{k_1,k_2}$-stable. The spectrum of the preconditioned matrix is clustered and therefore, the preconditioned GMRES method converges fast. Moreover, the operation cost in each iteration of the preconditioned GMRES method by using our BCCB preconditioner is less than that required by using block-circulant preconditioners proposed earlier. In numerical experiments, we compare the number of iterations of various preconditioners. Copyright © 2003 John Wiley & Sons, Ltd.

KEY WORDS: BVM; ODE; GMRES method; BCCB preconditioner; block-circulant preconditioner; Toeplitz matrix

1. INTRODUCTION TO ODES SOLVERS

In this paper, we consider the following linear initial value problem:

\[ y'(t) = J_m y(t) + g(t), \quad t \in (t_0, T] \]
\[ y(t_0) = z \quad (1) \]

where $y(t), g(t): \mathbb{R} \rightarrow \mathbb{R}^m$, $z \in \mathbb{R}^m$ and $J_m \in \mathbb{R}^{m \times m}$ is the Jacobian matrix. To approximate the solution of (1), we consider a generalization of the linear multistep technique known as the boundary value method (BVM), see, for instance, References [1, 2]. A BVM approximates the solution of (1) by means of a discrete boundary value problem. Let grid points be...

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