Circulant Preconditioners for Solving Differential Equations with Multidelays

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Abstract—We consider the solution of differential equations with multidelays by using boundary value methods (BVMs). These methods require the solution of some nonsymmetric, large and sparse linear systems. The GMRES method with the Strang-type block-circulant preconditioner is proposed to solve these linear systems. If an \( A_{k_1,k_2} \)-stable BVM is used, we show that our preconditioner is invertible and the spectrum of the preconditioned matrix is clustered. It follows that when the GMRES method is applied to solving the preconditioned systems, the method would converge fast. Numerical results are given to show the effectiveness of our methods. © 2004 Elsevier Ltd. All rights reserved.

Keywords—Differential equation with multidelays, BVM, Block-circulant preconditioner, GMRES method.

1. INTRODUCTION

In this paper, we consider the solution of a differential equation with multidelays

\[
\begin{align*}
    y'(t) &= J_n y(t) + D_n^{(1)} y(t - \tau_1) + \cdots + D_n^{(s)} y(t - \tau_s) + f(t), \quad t \geq t_0; \\
    y(t) &= \phi(t), \quad t \leq t_0;
\end{align*}
\]

(1)

by boundary value methods (BVMs), where \( y(t), f(t), \phi(t) : \mathbb{R} \to \mathbb{R}^n \); \( J_n, D_n^{(1)}, \ldots, D_n^{(s)} \in \mathbb{R}^{n \times n} \), and \( \tau_1, \ldots, \tau_s > 0 \) are some rational numbers. Such kind of equation appears in many applications [1,2]. The BVMs that we used are relatively new numerical methods for solving ordinary differential equations (ODEs), which is based on the linear multistep formulae, see [1]. The advantage in using BVMs over classical initial value methods (IVMs) comes from the stability properties of BVMs although IVMs, where the system of equations can be solved easily

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